Resoluções

Potenciação

Agora é com você – Pág. 6

01 a) $8 \cdot 8 \cdot 8 = 512$

b) $(-4) \cdot (-4) \cdot (-4) \cdot (-4) = 256$

c) 0 }

d) 1 } Consequências de definição

e) $(-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) = 1$

Agora é com você – Pág. 7

01 a) $\underbrace{(-1) \cdot (-1) \cdot ... \cdot (-1)}_{11 \text{ vezes}} = -1$

b) $\dfrac{1}{-6^2} = \dfrac{1}{-36} = -\dfrac{1}{36}$

c) $\dfrac{1}{(-6)^2} = \dfrac{1}{36}$

d) $\left(\dfrac{13}{10}\right)^2 = \dfrac{169}{100} = 1{,}69$

e) $-\left(-\dfrac{3}{2}\right)^4 = -\left(+\dfrac{81}{16}\right) = -\dfrac{81}{16}$

Agora é com você – Pág. 7

01 a) $3{,}84 \cdot 100\,000 = 3{,}84 \cdot 10^5$

b) $\dfrac{9{,}11}{\underset{31\,\text{zeros}}{100...00}} = 9{,}11 \cdot 10^{-31}$

c) $5{,}974 \cdot \underset{24\ \text{zeros}}{100...0} = 5{,}974 \cdot 10^{24}$

d) $\dfrac{1{,}67}{\underset{27\,\text{zeros}}{100...0}} = 1{,}67 \cdot 10^{-27}$

e) $3 \cdot 100\,000\,000 = 3 \cdot 10^8$

Agora é com você – Pág. 8

01 a) $\dfrac{(-5)^3 \cdot (-5)^{38}}{[(-5)^2]^{20}} = \dfrac{(-5)^{41}}{(-5)^{40}} = (-5)^1$

b) $\dfrac{(-2)^{50} \cdot (-10)^{50}}{4^{37} \cdot 5^{37}} = \dfrac{[(-2) \cdot (-10)]^{50}}{(4 \cdot 5)^{37}} = \dfrac{(20)^{50}}{(20)^{37}} = 20^{13}$

c) $\dfrac{(0{,}2)^{21}}{(5^{-1})^{-15}} = \dfrac{\left(\dfrac{2}{10}\right)^{21}}{\left(\dfrac{1}{5}\right)^{-15}} = \dfrac{\left(\dfrac{1}{5}\right)^{21}}{\left(\dfrac{1}{5}\right)^{-15}} = \left(\dfrac{1}{5}\right)^{36} = 5^{-36}$

d) $[(x^3)^{-2}]^{-n} = x^{6n}$

Agora é com você – Pág. 10

01 **A**

$4 \cdot 2^6 - 4^4$

$2^2 \cdot 2^6 - (2^2)^4$

$2^8 - 2^8 = 0$

02 $x = (-1)^{8n} - (-1)^{4n+7} + (-1)^{7n} - 2 \cdot (-1)^n$

Considerando **n** par:

$x = (+1) - (-1) + (+1) - 2 \cdot (+1) = 1 + 1 + 1 - 2 = 1$

Considerando **n** ímpar:

$x = (+1) - (-1) + (-1) - 2 \cdot (-1) = 1 + 1 - 1 + 2 = 3$

TESTANDO SEUS CONHECIMENTOS

01 a) $3^{3p} \cdot 3^{-2p} = 3^{3p+(-2p)} = 3^p$

b) $(-2)^4 \cdot (-2) \cdot \left(\dfrac{1}{2^{-6}}\right) = (-2)^4 \cdot (-2)^1 \cdot (2)^6 = (-2)^5 \cdot (2)^6 = -2^{11}$

c) $5^{2x+1} : 5^{x+1} = 5^{2x+1-(x+1)} = 5^{2x-x+1-1} = 5^x$

d) $[4^5(8^2 \cdot 125)^{14}] : (10^5 \cdot 2^{11} \cdot 25^{-1})^4 =$

$= [2^{10} \cdot (2^6 \cdot 5^3)^{14}] : (10^{20} \cdot 2^{44} \cdot 5^{-8}) =$

$= [2^{10} \cdot 2^{84} \cdot 5^{42}] : (10^{20} \cdot 2^{44} \cdot 5^{-8}) = \dfrac{2^{94} \cdot 5^{42}}{2^{20} \cdot 5^{20} \cdot 2^{44} \cdot 5^{-8}} =$

$= \dfrac{2^{94} \cdot 5^{42}}{2^{64} \cdot 5^{12}} = 2^{30} \cdot 5^{30} = (2 \cdot 5)^{30} = 10^{30}$

e) $\dfrac{[(a^{-3})^p]^2}{[a^4 \cdot a^{-10}]^5} = \dfrac{a^{-6p}}{[a^{-6}]^5} = \dfrac{a^{-6p}}{a^{-30}} = a^{-6p+30} = a^{-6 \cdot (p-5)}$

02 $E = \dfrac{x^3y^3 - x^4y^3}{x^3y^3 + x^3y^4} = \dfrac{x^3y^3 \cdot (1-x)}{x^3y^3 \cdot (1+y)} = \dfrac{1-x}{1+y}$

$E = \dfrac{1 - \left(\dfrac{1}{2}\right)^{-1}}{1 + (-3)} = \dfrac{1-2}{1-3} = \dfrac{-1}{-2} = \dfrac{1}{2}$

03 **C**

$\dfrac{50^{50}}{(5^2)^{25}} = \dfrac{50^{50}}{5^{50}} = 10^{50} = (10^2)^{25} = 100^{25}$

04 a) $\dfrac{2 \cdot 10^{-7} + 3 \cdot 2 \cdot 10^{-7}}{10^{-7} + 2 \cdot 10^{-7}} = \dfrac{8 \cdot 10^{-7}}{3 \cdot 10^{-7}} = \dfrac{8}{3}$

b) $\dfrac{(10^{-3})^3 \cdot [(10^{-3})^2]^{-3}}{(10^{-2})^3 \cdot (10^{-4})^4} = \dfrac{10^{-9} \cdot 10^{18}}{10^{-6} \cdot 10^{-16}} = \dfrac{10^9}{10^{-22}} = 10^{31}$

05 $\dfrac{-0{,}25^4 \cdot (-0{,}125^2)^{-3} \cdot [-(-0{,}0625)^{-2}]^3}{-(-0{,}5)^{-3} \cdot 0{,}03125^{-5}} =$

$= \dfrac{-[(0{,}5)^2]^4 \cdot \{-[(0{,}5)^3]^2\}^{-3} \cdot \{-[-(0{,}5)^4]^{-2}\}^3}{-(-0{,}5)^{-3} \cdot [(0{,}5)^5]^{-5}} =$

$= \dfrac{-(0{,}5)^8 \cdot [-(0{,}5)^{-18}] \cdot [-(0{,}5)^{-24}]}{(0{,}5)^{-3} \cdot (0{,}5)^{-25}} =$

$= \dfrac{-(0{,}5)^{-34}}{(0{,}5)^{-28}} = -(0{,}5)^{-6} = -2^6 = -64$

06 $\dfrac{(a-b)^3}{(a-b)^2} = a - b$

07 a) $3^x = \dfrac{1}{81}$

$3^x = \dfrac{1}{3^4} = 3^{-4}$

$x = -4$

b) $10^x = 1\,000\,000$

$10^x = 10^6$

$x = 6$

c) $5^x = 0{,}2 = \dfrac{2}{10} = \dfrac{1}{5}$

$5^x = 5^{-1} \Rightarrow x = -1$

d) $10^x = 0{,}00000001$

$10^x = 10^{-8}$

$x = -8$

e) $(0{,}001)^x = \dfrac{1}{10^{93}}$

$(10^{-3})^x = 10^{-93}$

$10^{-3x} = 10^{-93}$

$-3x = -93$

$x = 31$

08 $\dfrac{2^x \cdot (1 + 2 - 5 \cdot 2^2)}{2^x \cdot (1 - 2^3 - 17)} = \dfrac{3 - 20}{1 - 8 - 17} = \dfrac{-17}{-24} = \dfrac{17}{24}$

✎ ATIVIDADES PROPOSTAS

01 $m = \dfrac{10^{-5} \cdot 10^{-6} \cdot 10^5}{10^{-3} \cdot (10^{-1})^3} = \dfrac{10^{-6}}{10^{-6}} = 1 \to m^2 = (1)^2 = 1$

02 $\dfrac{x^{-1} + y^{-1}}{(xy)^{-1}} = \dfrac{\dfrac{1}{x} + \dfrac{1}{y}}{\dfrac{1}{xy}} = \dfrac{\dfrac{y+x}{xy}}{\dfrac{1}{xy}} = \dfrac{y+x}{xy} \cdot \dfrac{xy}{1} = y + x$

03 $[2^{-1} + (-2)^{-1} - (-2)^2] : (1 - 2^{-2}) + \left(\dfrac{3}{16}\right)^{-1} + \left(\dfrac{1}{1024}\right)^{-1}$

$\left[\dfrac{1}{2} - \dfrac{1}{2} - 4\right] : \left(1 - \dfrac{1}{4}\right) + \dfrac{16}{3} + 1024$

$[-4] : \left(\dfrac{4-1}{4}\right) + \dfrac{16}{3} + 1024$

$[-4] : \left(+\dfrac{3}{4}\right) + \dfrac{16}{3} + 1024$

$[-4] \cdot \left(+\dfrac{4}{3}\right) + \dfrac{16}{3} + 1024 = -\dfrac{16}{3} + \dfrac{16}{3} + 1024 = 1024$

04 **C**

$2^x = b$

$2^{-2+2x} = 2^{-2} \cdot 2^{2x} = \dfrac{1}{4} \cdot (2^x)^2 = \dfrac{1}{4} \cdot (b)^2 = \dfrac{1}{4} \cdot b^2 = \dfrac{b^2}{4}$

05 $1 + 2 + 1 = 4 = 2^2$

$1 + 2 + 3 + 2 + 1 = 9 = 3^2$

$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16 = 4^2$

$A = 1 + 2 + 3 + \ldots + 2008 + 2009 + 2008 + \ldots + 1 = 2009^2$

$\dfrac{A}{287^2} = \dfrac{(2009)^2}{(287)^2} = \left(\dfrac{2009}{287}\right)^2 = 7^2 = 49$

06 **C**

$\dfrac{2^{2003} \cdot 3^{2002}}{2^{2002} \cdot 3^{2003}} + \dfrac{2^{2002} \cdot 3^{2002}}{2^{2002} \cdot 3^{2003}}$

$2 \cdot \dfrac{1}{3} + 1 \cdot \dfrac{1}{3} = \dfrac{2}{3} + \dfrac{1}{3} = 1$

07 $\left[\dfrac{2^{2005} \cdot (2^2 + 1)}{2^{2004} \cdot (2^2 + 1)}\right] \cdot 2160$

$2 \cdot 2160 = 4320$

Soma dos algarismos: $4 + 3 + 2 + 0 = 9$.

08 **E**

$4^{16} : 4^4 = 4^{12}$

09 $E = \dfrac{(a-b)^3}{(a-b)^2} = a - b = 5 \cdot 10^8 - 2 \cdot 10^6 = 5 \cdot 10^2 \cdot 10^6 - 2 \cdot 10^6 =$

$= 500 \cdot 10^6 - 2 \cdot 10^6 = (500 - 2) \cdot 10^6 = 498 \cdot 10^6 = 4,98 \cdot 10^8$

10 $\dfrac{2^n \cdot (2^4 + 2^2 + 2^{-1})}{2^n \cdot (2^{-2} + 2^{-1})} = \dfrac{20 + \dfrac{1}{2}}{\dfrac{1}{4} + \dfrac{1}{2}} = \dfrac{\dfrac{40+1}{2}}{\dfrac{1+2}{4}} = \dfrac{\dfrac{41}{2}}{\dfrac{3}{4}} = \dfrac{41}{\cancel{2}} \cdot \dfrac{\cancel{4}^2}{3} = \dfrac{82}{3}$

11 **E**

$\dfrac{\left(\dfrac{1}{2}\right)^{x+4} - \dfrac{1}{2} \cdot \left(\dfrac{1}{2}\right)^x}{2^{-x} \cdot 2^{-4}} = \dfrac{\cancel{\left(\dfrac{1}{2}\right)^x} \cdot \left[\left(\dfrac{1}{2}\right)^4 - \left(\dfrac{1}{2}\right)\right]}{\cancel{\left(\dfrac{1}{2}\right)^x} \cdot \left(\dfrac{1}{2}\right)^4} =$

$= \dfrac{\dfrac{1}{16} - \dfrac{1}{2}}{\dfrac{1}{16}} = \dfrac{\dfrac{1-8}{16}}{\dfrac{1}{16}} = \dfrac{\dfrac{-7}{16}}{\dfrac{1}{16}} = \dfrac{-7}{\cancel{16}} \cdot \dfrac{\cancel{16}}{1} = -7$

MERGULHANDO FUNDO

01 **E**

$x = 0,00375 \cdot 10^{-6}$ e $y = 22,5 \cdot 10^{-8}$

$x = 3,75 \cdot 10^{-9}$ e $y = 225 \cdot 10^{-9}$

$\dfrac{y}{x} = \dfrac{225 \cdot \cancel{10^{-9}}}{3,75 \cdot \cancel{10^{-9}}} = 60$. Logo, $y = 60x$.

02 $2^x = 2^{3y+3}$ $\qquad\qquad 3^{2y} = 3^{x-9}$

$x = 3y + 3$ $\qquad\qquad 2y = x - 9$

$\begin{cases} x = 3y + 3 \\ 2y = x - 9 \end{cases}$

Utilizando o método da substituição, tem-se:

$2y = 3y + 3 - 9 \qquad\qquad x = 3 \cdot 6 + 3$

$-y = -6 \cdot (-1) \qquad\qquad x = 21$

$y = 6$

$x + y = 21 + 6 = 27$

03 $\left\{\left[\dfrac{(-10^4)^9}{(10^{-2})^{-27}} : (-(10^{-1})^9)^2\right] : (10^{-3})^{-4}\right\} : [(10^3)^4]^{-1}$

$= \left\{\left[\dfrac{-10^{36}}{10^{54}} : 10^{-18}\right] : 10^{12}\right\} : 10^{-12} =$

$= \left\{\left[-10^{-18} : 10^{-18}\right] : 10^{12}\right\} : 10^{-12} =$

$= \left\{-1 : 10^{12}\right\} : 10^{-12} = -10^{-12} : 10^{-12} = -1$

04 I. $P = 3^2 \cdot 7 \cdot 10^{50} \cdot 2^{31} \cdot 5^{40} \cdot 2^6$

$P = 3^2 \cdot 7 \cdot 10^{50} \cdot 2^{37} \cdot 5^{37} \cdot 5^3$

$P = 3^2 \cdot 5^3 \cdot 7 \cdot 10^{50} \cdot (2 \cdot 5)^{37}$

$P = 3^2 \cdot 5^3 \cdot 7 \cdot 10^{50} \cdot 10^{37}$

$P = 3^2 \cdot 5^3 \cdot 7 \cdot 10^{87}$

Logo, o produto P termina em 87 zeros.

II. $A^2 - B^2 = (A + B) \cdot (A - B)$

$\left(\dfrac{3^x + 3^{-x}}{2} + \dfrac{3^x - 3^{-x}}{2}\right) \cdot \left(\dfrac{3^x + 3^{-x}}{2} - \dfrac{3^x - 3^{-x}}{2}\right) \Rightarrow$

$3^x \cdot 3^{-x} = 3^0 = 1$

III. a) $3^{n+2} \cdot 2^{n+3} = 2592 = 2^5 \cdot 3^4$

$n + 3 = 5$ ou $n + 2 = 4 \Rightarrow n = 2$

b) $3^n \cdot (1 + 3 + 9 + 27) = 1080$

$3^n \cdot 40 = 1080 \Rightarrow 3^n = \dfrac{1080}{40} \Rightarrow 3^n = 27 = 3^3 \Rightarrow n = 3$