Resoluções

Capítulo 12

Relações métricas na circunferência

Agora é com você – Pág. 26

01 $R = 4$ cm

$C = 2\pi R = 2\pi \cdot 4 = 8\pi$ cm

02 $C = 2\pi R$

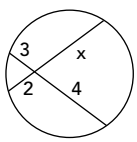
$2\pi \cdot R = 12{,}56 \Rightarrow 2 \cdot 3{,}14 \cdot R = 12{,}56$

$6{,}28R = 12{,}56$

$R = \dfrac{12{,}56}{6{,}28} = 2$ cm

Agora é com você – Pág. 27

01 a)



$2x = 3 \cdot 4$

$2x = 12$

$x = 6$

b) $8 \cdot 6 = x \cdot (4 + x + 4)$

$48 = x(8 + x)$

$48 = 8x + x^2$

$x^2 + 8x - 48 = 0$

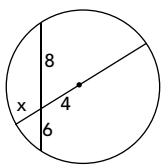
$\Delta = 8^2 - 4 \cdot 1 \cdot (-48)$

$\Delta = 64 + 192 = 256$

$x = \dfrac{-8 \pm 16}{2}$

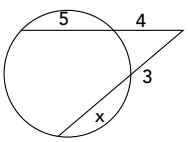
$x' = 4$

$x'' = -12$ (não serve)



Agora é com você – Pág. 29

01



$4 \cdot \cancel{9}^{3} = \cancel{3}(3 + x)$

$12 = 3 + x$

$x = 9$

Agora é com você – Pág. 30

01

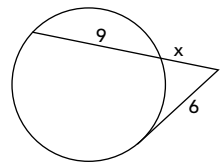
$6^2 = x(9 + x) \Rightarrow 36 = x^2 + 9x$

$x^2 + 9x - 36 = 0$

$\Delta = 9^2 - 4 \cdot 1(-36) \Rightarrow \Delta = 225$

$x = \dfrac{-9 \pm 15}{2} \Rightarrow x' = \dfrac{6}{2} = 3$

$x'' = -12$ (não serve)



TESTANDO SEUS CONHECIMENTOS

01 $D = 2R$

$D = 1$ m

$2R = 1 \Rightarrow R = 0{,}50$ m

$C = 2\pi R = 2 \cdot 3{,}14 \cdot 0{,}5 = 2 \cdot \dfrac{314}{100} \cdot \dfrac{5}{10} = \dfrac{314}{100}$

1 volta — $\dfrac{314}{100}$

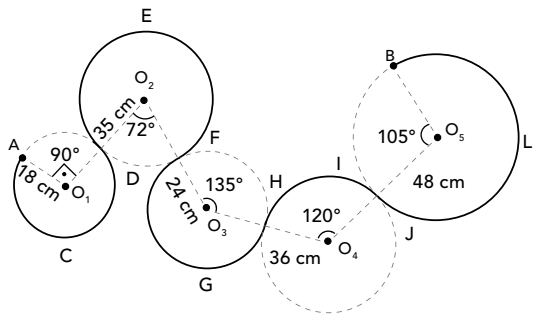
x — 62800

$\dfrac{314}{100}x = 62800$

$x = \dfrac{6280000}{314}$

$x = 20000$ voltas

02



$\overset{\frown}{ACD} = \left(\dfrac{270°}{360°}\right) \cdot 2\pi \cdot 18 = 27\pi$ cm

$\overset{\frown}{DEF} = \left(\dfrac{288°}{360°}\right) \cdot 2\pi \cdot 35 = 56\pi$ cm

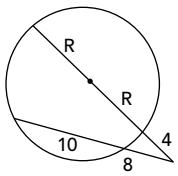
$\overset{\frown}{FGH} = \left(\dfrac{225°}{360°}\right) \cdot 2\pi \cdot 24 = 30\pi$ cm

$\overset{\frown}{HIJ} = \left(\dfrac{120°}{360°}\right) \cdot 2\pi \cdot 36 = 24\pi$ cm

$\overset{\frown}{JLB} = \left(\dfrac{255°}{360°}\right) \cdot 2\pi \cdot 48 = 68\pi$ cm

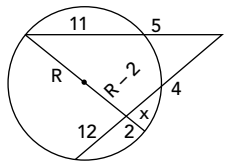
Portanto:
$\overline{ACD} + \overline{DEF} + \overline{FGH} + \overline{HIJ} + \overline{JLB} = 205\pi$ cm

03 a)



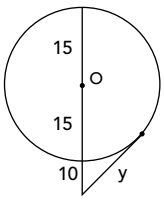
$\overset{2}{\cancel{8}} \cdot 18 = \cancel{4} \cdot (4 + 2R)$
$36 = 4 + 2R$
$32 = 2R$
$R = 16$

b)



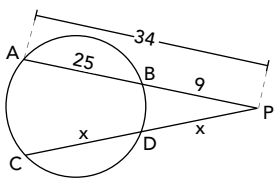
$5 \cdot \overset{4}{\cancel{16}} = \cancel{4} \cdot (16 + x)$
$20 = 16 + x \Rightarrow x = 4$
$\overset{2}{\cancel{4}} \cdot 12 = \cancel{2} \cdot (2R - 2)$
$24 = 2R - 2 \Rightarrow 2R = 26$
$R = 13$

04 **C**



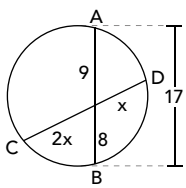
$y^2 = 10 \cdot 40$
$y^2 = 400$
$y = 20$

05 a)



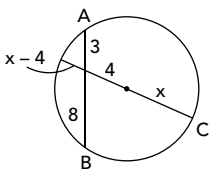
$x \cdot 2x = 9 \cdot 34$
$\cancel{2}x^2 = 9 \cdot \overset{17}{\cancel{34}}$
$x = \sqrt{9 \cdot 17}$
$x = 3\sqrt{17}$

b)



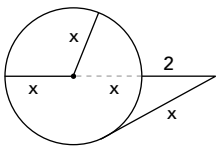
$2x \cdot x = 9 \cdot 8$
$2x^2 = 72$
$x^2 = 36$
$x = 6$

c)



$3 \cdot 8 = (x - 4) \cdot (x + 4)$
$24 = x^2 - 16$
$x^2 = 40$
$x = \sqrt{40} = 2\sqrt{10}$

d)



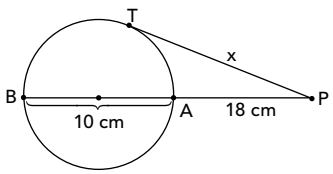
$x^2 = 2(2x + 2)$
$x^2 = 4x + 4$
$x^2 - 4x - 4 = 0$
$\Delta = (-4)^2 - 4 \cdot 1 \cdot (-4) = 16 + 16 = 32$
$x = \dfrac{4 \pm 4\sqrt{2}}{2} \Rightarrow \begin{cases} x' = 2 + 2\sqrt{2} \\ x'' = 2 - 2\sqrt{2} \text{ (não serve)} \end{cases}$

ATIVIDADES PROPOSTAS

01 a) Sejam C, c_1, c_2, c_3, respectivamente, os comprimentos "normal", aumentando o raio em 2 m, aumentando o raio em 3 m e aumentando o raio em **a** metros. Tem-se:
$C = 2\pi R$
$c_1 = 2\pi(R + 2) \Rightarrow c_1 = 2\pi R + 4\pi \Rightarrow c_1 = C + 4\pi$
$c_2 = 2\pi(R + 3) \Rightarrow c_2 = 2\pi R + 6\pi \Rightarrow c_2 = C + 6\pi$
$c_3 = 2\pi(R + a) \Rightarrow c_3 = 2\pi R + 2a\pi \Rightarrow c_3 = C + 2a\pi$

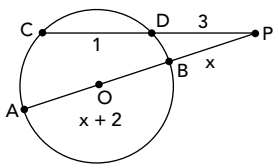
Portanto, o aumento no comprimento será de 4π m, 6π m e $2a\pi$ m, respectivamente.

b)



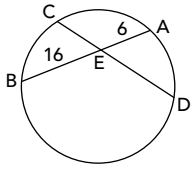
$x^2 = 18 \cdot 28$
$x = \sqrt{18 \cdot 28}$
$x = \sqrt{9 \cdot 2 \cdot 4 \cdot 7}$
$x = 6\sqrt{14}$ cm

02



$3 \cdot 4 = x(2x + 2)$
$\cancel{12} = \cancel{2x}(x + 1)$
$6 = x(x + 1)$
$x^2 + x - 6 = 0$
$\Delta = b^2 - 4ac$
$\Delta = 1^2 - 4 \cdot 1 \cdot (-6) = 25$
$x = \dfrac{-1 \pm 5}{2}$
$x' = \dfrac{4}{2} = 2$
$x'' = \dfrac{-6}{2} = -3$ (não serve)
$\overline{PA} = 2x + 2 = 2 \cdot 2 + 2 = 6$

03



$\frac{\overline{ED}}{\overline{EC}} = \frac{2}{3} \Rightarrow \overline{ED} = \frac{2}{3}\overline{EC}$

$\overline{EA} \cdot \overline{EB} = \overline{EC} \cdot \overline{ED}$

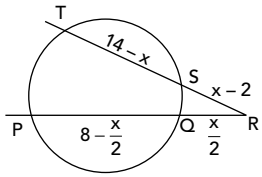
$6 \cdot 16 = \overline{EC} \cdot \overline{ED}$

$\cancel{96}^{48} = \frac{\cancel{2}}{3}\overline{EC}^2 \Rightarrow \overline{EC}^2 = 144 \Rightarrow \overline{EC} = 12$

$\overline{ED} = \frac{2}{\cancel{3}} \cdot \overset{4}{\cancel{12}} = 8$

Portanto, $\overline{CD} = 12 + 8 = 20$

04



$(x-2) \cdot 12 = \frac{x}{2} \cdot 8$

$(x-2) \cdot \overset{3}{\cancel{12}} = \cancel{4}x$

$(x-2) \cdot 3 = x$

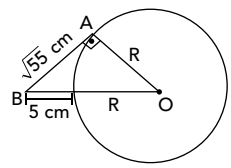
$3x - 6 = x$

$3x - x = 6$

$2x = 6 \Rightarrow x = 3$

$\overline{RS} = x - 2 = 3 - 2 = 1$ cm

05



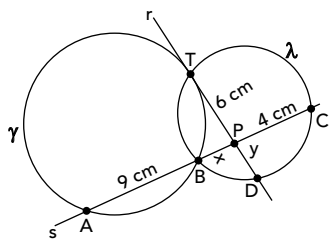
$(5+R)^2 = R^2 + (\sqrt{55})^2$

$25 + 10R + \cancel{R^2} = \cancel{R^2} + 55$

$10R = 30 \Rightarrow R = 3$

$C = 2\pi R \Rightarrow C = 6\pi$ cm

06 a)



$6^2 = x(9+x)$

$36 = 9x + x^2$

$x^2 + 9x - 36 = 0$

$\Delta = 9^2 - 4 \cdot 1 \cdot (-36) = 81 + 144 = 225$

$x = \frac{-9 \pm 15}{2} \Rightarrow \begin{cases} x' = \frac{6}{2} = 3 \\ x'' = -\frac{24}{2} = -12 \text{ (não serve)} \end{cases}$

$\overline{PB} = 3$ cm

b) $y \cdot 6 = 4 \cdot 3$

$6y = 12 \Rightarrow y = \frac{12}{6} = 2$ cm

$\overline{PD} = 2$ cm

07

$3(3+x) = 2(2+3x)$

$9 + 3x = 4 + 6x$

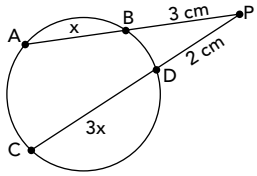
$5 = 3x$

$x = \frac{5}{3}$

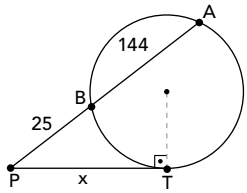
$\overline{PC} = 2 + 3x$

$\overline{PC} = 2 + \cancel{3} \cdot \frac{5}{\cancel{3}}$

$\overline{PC} = 7$ cm



08 a)



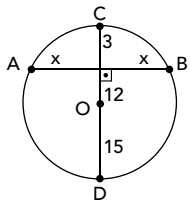
$x^2 = 25 \cdot 169$

$x = \sqrt{25 \cdot 169}$

$x = 5 \cdot 13$

$x = 65$

b)

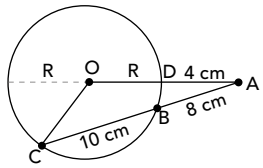


$x^2 = 3 \cdot 27$

$x^2 = 81$

$x = 9$

09



$\overset{2}{\cancel{8}} \cdot 18 = \cancel{4}(4 + 2R)$

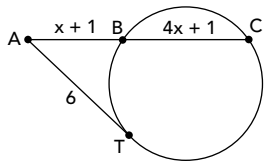
$36 = 4 + 2R$

$R = 16$

Portanto, o perímetro do triângulo pedido é:

$2p = 16 + 18 + 20 = 54$ cm

10



$(x + 1 + 4x + 1) \cdot (x + 1) = 6^2$

$(5x + 2) \cdot (x + 1) = 36$

$5x^2 + 7x + 2 - 36 = 0$

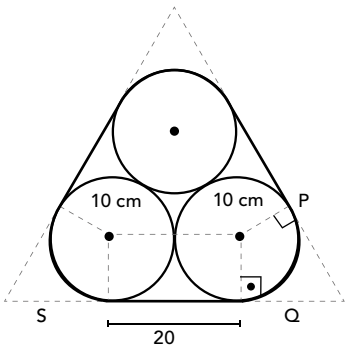
$5x^2 + 7x - 34 = 0$

$\Delta = 7^2 - 4 \cdot 5 \cdot (-34) = 49 + 680 = 729$

$x = \dfrac{-7 \pm 27}{10} \Rightarrow \begin{cases} x' = 2 \\ x'' = -\dfrac{17}{5} \text{ (não serve)} \end{cases}$

MERGULHANDO FUNDO

01



$PQ = \dfrac{1}{3} \cdot 2\pi \cdot R = \dfrac{1}{3} \cdot 2\pi \cdot 10 = \dfrac{20}{3}\pi$ cm

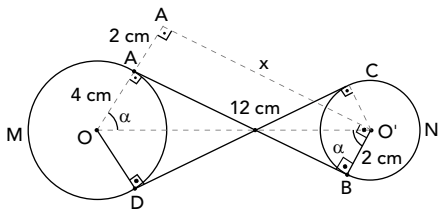
Também tem-se:

$QS = 2R \Rightarrow QS = 2 \cdot 10 = 20$ cm

Logo, o comprimento da correia será dado por:

$3\left(20 + \dfrac{20}{3}\pi\right) = 60 + 20\pi = 20(3 + \pi)$ cm

02



$\Delta AOO' \Rightarrow 12^2 = 6^2 + x^2$

$144 = 36 + x^2 \Rightarrow x^2 = 108 \Rightarrow x = \sqrt{108} = 6\sqrt{3}$ cm

$\overline{AB} = 6\sqrt{3}$ cm, $\overline{CD} = 6\sqrt{3}$ cm

Seja $A\hat{O}O' = \alpha$. Tem-se:

$\cos\alpha = \dfrac{OA}{OO'} \Rightarrow \cos\alpha = \dfrac{6}{12} = \dfrac{1}{2} \therefore \alpha = 60° \therefore A\hat{O}D = 120°$

$A\hat{O}O'$ e $B\hat{O}'O$ são alternos $\Rightarrow B\hat{O}'O = \alpha = 60° \Rightarrow B\hat{O}'C = 120°$

$\overset{\frown}{AMD} = \dfrac{240°}{360°} \cdot 2\pi \cdot 4 = \dfrac{16\pi}{3}$ cm

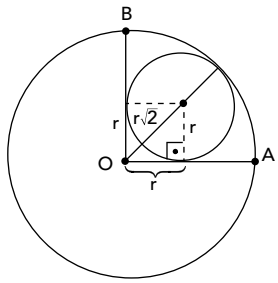
$\overset{\frown}{BNC} = \dfrac{240°}{360°} \cdot 2\pi \cdot 2 = \dfrac{8\pi}{3}$ cm

Logo, o comprimento da corrente será dado por:

$\overline{AB} + \overline{CD} + \overset{\frown}{AMD} + \overset{\frown}{BNC} = 6\sqrt{3} + 6\sqrt{3} + \dfrac{16\pi}{3} + \dfrac{8\pi}{3}$

$C = 12\sqrt{3} + 8\pi \Rightarrow C = 4 \cdot (3\sqrt{3} + 2\pi)$ cm.

03

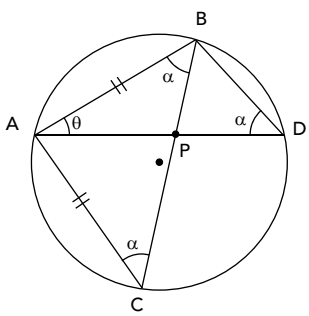


$r\sqrt{2} + r = R$

$r(\sqrt{2} + 1) = R$

$r = \dfrac{R}{(\sqrt{2} + 1)} \cdot \dfrac{(\sqrt{2} - 1)}{(\sqrt{2} - 1)} = \dfrac{R(\sqrt{2} - 1)}{(\sqrt{2})^2 - 1^2} = R(\sqrt{2} - 1)$

04



$\Delta ABD \sim \Delta APB$

$\overline{AC} \cong \overline{AB}$

ΔABC isósceles

$\alpha \cong A\hat{B}C \cong A\hat{C}B$

$A\hat{D}B \cong \alpha$ (inscrito em $\overset{\frown}{AB}$)

ΔABD (θ, α)

ΔAPB (θ, α)

↓

Ângulo comum

Portanto, ΔABD e ΔAPB são semelhantes.