Resoluções

Capítulo 12

Transformações trigonométricas – Arco duplo

ATIVIDADES PARA SALA

01 $\text{sen } a = \frac{1}{5} \qquad \cos a = -\frac{2\sqrt{6}}{5} \qquad \text{tg } a = -\frac{\sqrt{6}}{12}$

a) $\text{sen } 2a = 2 \cdot \frac{1}{5} \cdot \left(-\frac{2\sqrt{6}}{5}\right) = -\frac{4\sqrt{6}}{25}$

b) $\cos 2a = \left(-\frac{2\sqrt{6}}{5}\right)^2 - \left(\frac{1}{5}\right)^2 = \frac{23}{25}$

c) $\text{tg } 2a = \frac{\text{sen } 2a}{\cos 2a} = \frac{-\frac{4\sqrt{6}}{25}}{\frac{23}{25}} = -\frac{4\sqrt{6}}{23}$

02 a) 2mn

b) $n^2 - m^2$ ou $2n^2 - 1$ ou $1 - 2m^2$

c) $\frac{\frac{2m}{n}}{1 - \frac{m^2}{n^2}} = \frac{2m}{n} \cdot \frac{n^2}{n^2 - m^2} = \frac{2mn}{n^2 - m^2}$

03 $\cos x = -\frac{1}{4} \qquad \text{sen } x = -\frac{\sqrt{15}}{4} \qquad \text{tg } x = \sqrt{15}$

$\text{sen } 2x = 2 \cdot \left(-\frac{\sqrt{15}}{4}\right)\left(-\frac{1}{4}\right) = \frac{\sqrt{15}}{8}$

$\cos 2x = \frac{1}{16} - \frac{15}{16} = -\frac{7}{8}$

$\text{tg } 2x = \frac{2\sqrt{15}}{1 - 15} = -\frac{\sqrt{15}}{7}$

04 $\frac{\text{sen } x}{\cos x} + \frac{\cos x}{\text{sen } x} = 3 \Rightarrow \text{sen}^2 x + \cos^2 x = 3 \text{ sen } x \cdot \cos x \Rightarrow$

$\Rightarrow \text{sen } x \cdot \cos x = \frac{1}{3} \Rightarrow \text{sen } 2x = \frac{2}{3}$

05 **A**

$(\text{sen } x + \cos x)^2 - \text{sen } 2x =$

$= \text{sen}^2 x + 2 \text{ sen } x \cdot \cos x + \cos^2 x - \text{sen } 2x =$

$= 1 + \text{sen } 2x - \text{sen } 2x = 1$

ATIVIDADES PROPOSTAS

01 **B**

Se $\text{sen } x = \frac{1}{2}$, então, $\cos x = \frac{\sqrt{3}}{2}$.

Se $y = \frac{\pi}{2}$, então, $\text{sen } y = 1$ e $\cos y = 0$.

Logo,

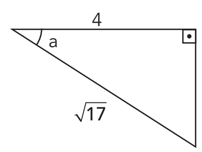
$\text{sen } (x - 2y) = \text{sen } x \cdot \cos 2y - \text{sen } 2y \cdot \cos x$

$\text{sen } (x - 2y) = \text{sen } x \cdot (\cos^2 y - \text{sen}^2 y) - (2 \text{ sen } y \cdot \cos y) \cdot \cos x.$

$\text{sen } (x - 2y) = \frac{1}{2} \cdot (0 - 1) - (2 \cdot 1 \cdot 0) \cdot \frac{\sqrt{3}}{2}$

$\text{sen } (x - 2y) = \frac{1}{2} \cdot (-1) = -\frac{1}{2} = -0{,}5$

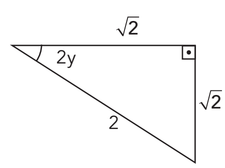
02 **B**



$\text{sen } 2a = 2 \cdot \frac{1}{\sqrt{17}} \cdot \frac{4}{\sqrt{17}} = \frac{8}{17}$

Observação: Se **a** for do 3º quadrante, sen a < 0 e cos a < 0. Mesmo assim, sen 2a > 0.

03 **A**



$\text{sen}^4 y + \cos^4 y = \frac{3}{4}$

$(\text{sen}^2 y + \cos^2 y)^2 - 2\text{sen}^2 y \cdot \cos^2 y = \frac{3}{4}$

$1 - 2(\text{sen } y \cdot \cos y)^2 = \frac{3}{4}$

$2(\text{sen } y \cdot \cos y)^2 = 1 - \frac{3}{4} \Rightarrow (\text{sen } y \cdot \cos y)^2 = \frac{1}{8}$

$\text{sen } y \cdot \cos y = \pm\frac{\sqrt{2}}{4} \Rightarrow 2\text{sen } y \cdot \cos y = \pm\frac{\sqrt{2}}{2} \Rightarrow \text{sen } 2y = \pm\frac{\sqrt{2}}{2}$

$\Rightarrow \text{tg } 2y = \pm 1$

04 **C**

$\text{sen } a + \cos a = \sqrt{2} \Rightarrow (\text{sen } a + \cos a)^2 = (\sqrt{2})^2 \Rightarrow$

$\Rightarrow 1 + \text{sen } 2a = 2 \Rightarrow \text{sen } 2a = 1 \Rightarrow 2a = 90° \Rightarrow$

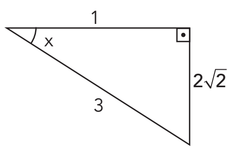
$\Rightarrow a = 45° \Rightarrow \text{tg } a = 1$

05 **E**

$\text{sen } x = -\frac{1}{2}$, $x \in 4^{\underline{o}}$ quadrante $\Rightarrow x = 330°$

$\text{sen } 4x = \text{sen } 1\,320° = \text{sen } 240° = -\frac{\sqrt{3}}{2}$

06 **E**



$$\cos 2x + \frac{1}{\sqrt{8}} \cdot \text{sen } 2x =$$
$$= \left(\frac{1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2 + \frac{1}{\cancel{\sqrt{8}}} \cdot 2 \cdot \frac{\cancel{2\sqrt{2}}}{3} \cdot \frac{1}{3} =$$
$$= \frac{1}{9} - \frac{8}{9} + \frac{2}{9} = -\frac{5}{9}$$

07 **A**

$$\frac{2\left(\text{sen } x \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cos x\right)\left(\cos x \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \text{sen } x\right)}{1 + \text{sen } 2x} =$$
$$= \frac{2\left(\frac{1}{2}\text{sen } x \cos x + \frac{1}{2}\text{sen}^2 x + \frac{1}{2}\cos^2 x + \frac{1}{2}\text{sen } x \cos x\right)}{1 + \text{sen } 2x} =$$
$$= \frac{2\left(\text{sen } x \cos x + \frac{1}{2}(\text{sen}^2 x + \cos^2 x)\right)}{1 + \text{sen } 2x} =$$
$$= \frac{2\left(\text{sen } x \cos x + \frac{1}{2}\right)}{1 + \text{sen } 2x} =$$
$$= \frac{2 \text{ sen } x \cos x + 1}{1 + \text{sen } 2x} = \frac{1 + \text{sen } 2x}{1 + \text{sen } 2x} = 1$$

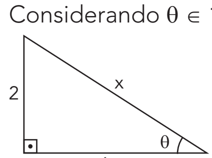
08 **B**

$$\cos 2x = \frac{1}{2} \Rightarrow 2\cos^2 x - 1 = \frac{1}{2} \Rightarrow 2\cos^2 x = \frac{3}{2} \Rightarrow$$
$$\Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow \text{sen } x = -\frac{1}{2}$$

09 **B**

$$\text{Sendo tg } \theta = 2 \Rightarrow \begin{cases} \theta \in 1^{\underline{o}}\text{ Q} \Rightarrow \text{sen } \theta > 0 \text{ e } \cos \theta > 0. \\ \text{ou} \\ \theta \in 3^{\underline{o}}\text{ Q} \Rightarrow \text{sen } \theta < 0 \text{ e } \cos \theta < 0. \end{cases}$$

- Considerando $\theta \in 1^{\underline{o}}$ quadrante, tem-se:



$$x^2 = 4 + 1$$
$$x^2 = 5$$
$$x = \sqrt{5}$$

$$\text{sen } \theta = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \Rightarrow \text{sen } \theta = \frac{2\sqrt{5}}{5}$$
$$\cos \theta = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \Rightarrow \cos \theta = \frac{\sqrt{5}}{5}$$

$$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos 2\theta = 2 \cdot \frac{5}{25} - 1 = \frac{2}{5} - 1 = -\frac{3}{5}.$$
$$1 + \text{sen } 2\theta = 1 + 2 \cdot \text{sen } \theta \cdot \cos \theta = 1 + 2 \cdot \frac{2\sqrt{5}}{5} \cdot \frac{\sqrt{5}}{5} = 1 + \frac{4}{5} = \frac{9}{5}$$

Então, $\frac{\cos 2\theta}{1 + \text{sen } 2\theta} = -\frac{3}{9} = -\frac{1}{3}$.

- Considerando $\boldsymbol{\theta} \in 3^{\underline{o}}$ quadrante, encontra-se o mesmo valor pedido no quociente.

10 Na expressão, elevam-se os dois membros ao quadrado:

$$(\text{sen } x + \cos x)^2 = \left(\frac{1}{3}\right)^2$$
$$\text{sen}^2 x + 2 \text{ sen } x \cos x + \cos^2 x = \frac{1}{9} \Rightarrow 1 + 2 \text{ sen } x \cos x = \frac{1}{9} \Rightarrow$$
$$\Rightarrow 2 \text{ sen } x \cos x = \frac{1}{9} - 1 \Rightarrow 2 \text{ sen } x \cos x = \frac{1-9}{9} = -\frac{8}{9}$$

Como 2 sen x cos x = sen 2x, tem-se que: $\text{sen } 2x = -\frac{8}{9}$.